Highly-Efficient Multi-Coil Wireless Power Transfer (WPT)

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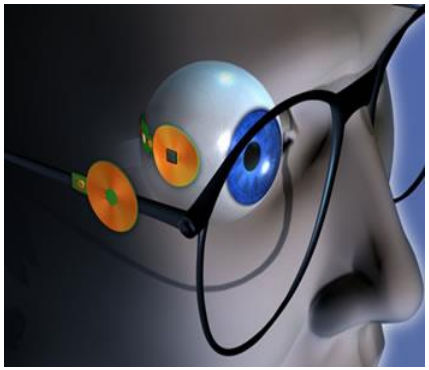
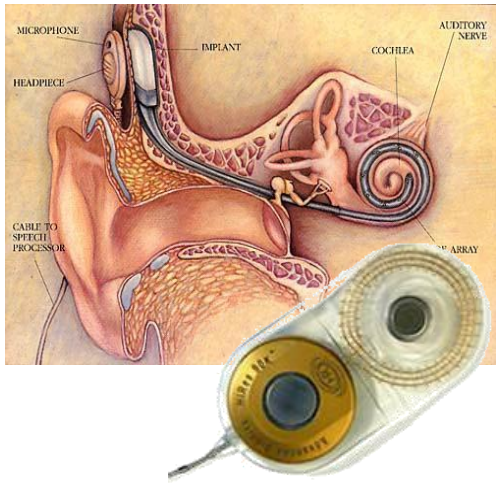
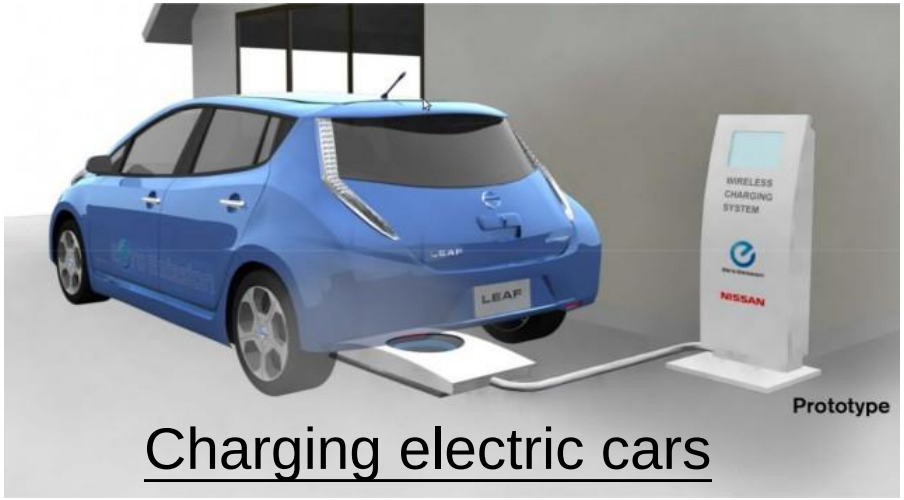
WPT Applications

Charging mobile electronics

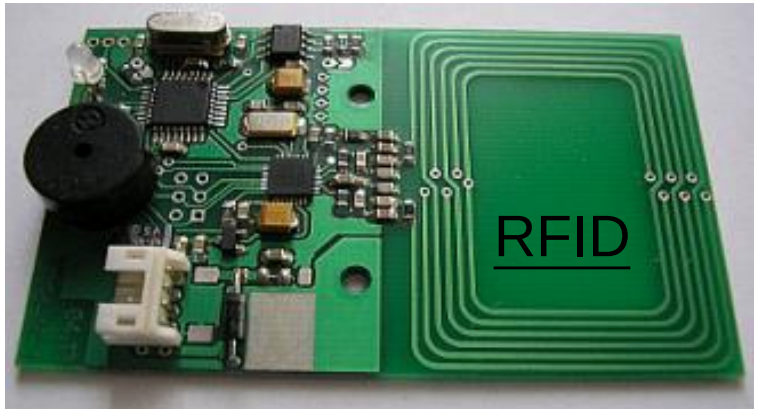




Implantable Medical Devices (IMDs)



Charging electric cars



Coupled-mode Magnetic Resonance-based Power Transmission

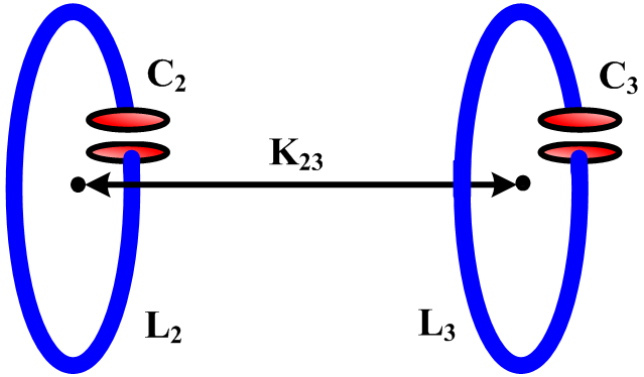
- Proposed by physicists at MIT based on **coupled-mode theory (CMT)**
- An alternative wireless power-transfer technique using typically **four** coils
- Goal: Increase the **power transfer efficiency (PTE)** at large coupling distances

60-W is transferred from 2-m away

A. Kurs, et. al, *Science* 2007

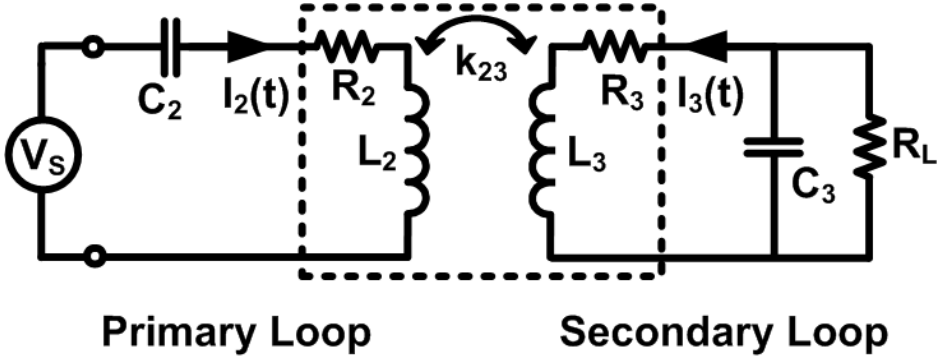
Questions?

- A need for a comprehensive circuit theory for coupled-mode magnetic resonance-based links
- A need for circuit theory based on the reflected-load theory (RLT) for multi-coil links
- Are coupled-mode magnetic resonance-based links always the optimal choice?
- A need for a new figure-of-merit (FoM) for high performance multi-coil design



Resonant-magnetic coupling

vs.



Inductive coupling

Calculating Power Transfer Efficiency (PTE) based on RLT

- Secondary $R_3L_3C_3$ can be reflected onto the primary:

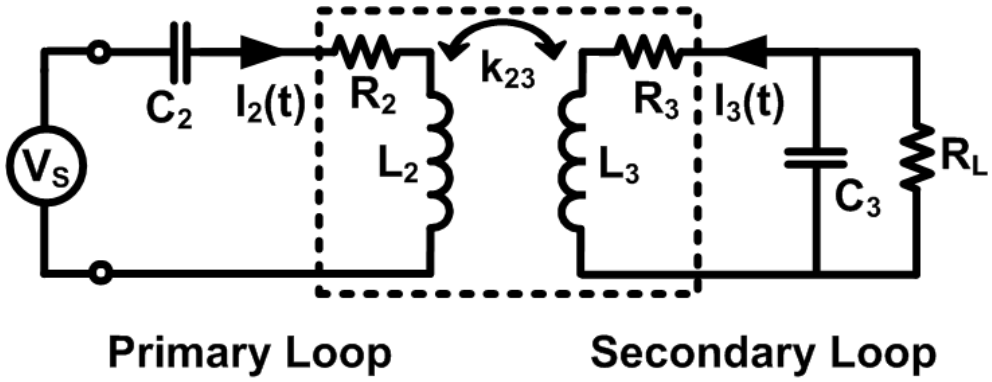
$$PTE = \frac{R_{ref}}{R_2 + R_{ref}} \frac{Q_3^2 R_3}{Q_3^2 R_3 + R_L} =$$

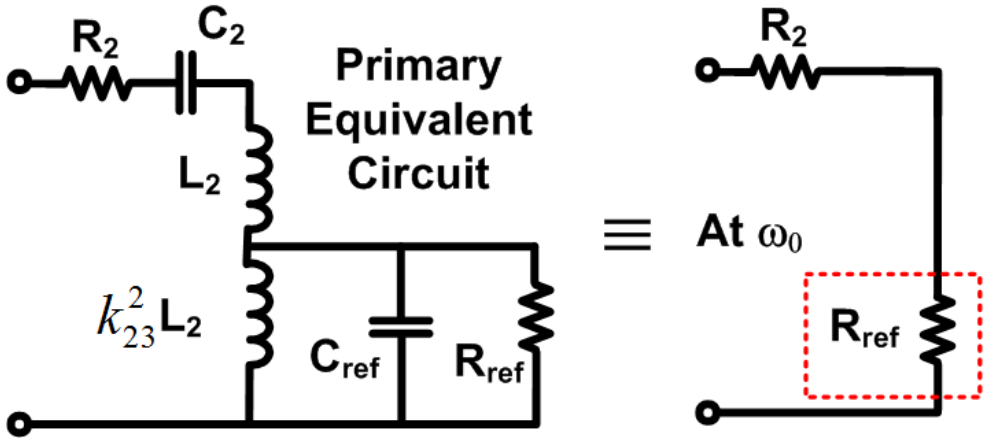
$$= \frac{k_{23}^2 Q_2 Q_{3L}}{1 + k_{23}^2 Q_2 Q_{3L}} \cdot \frac{Q_{3L}}{Q_L}$$

- PTE is highly dependent on: ***k_{23}***, ***Q_2***, and ***Q_3***

$Q_{3L} = Q_3 Q_L / (Q_3 + Q_L)$

$Q_L = R_L / \omega_0 L$

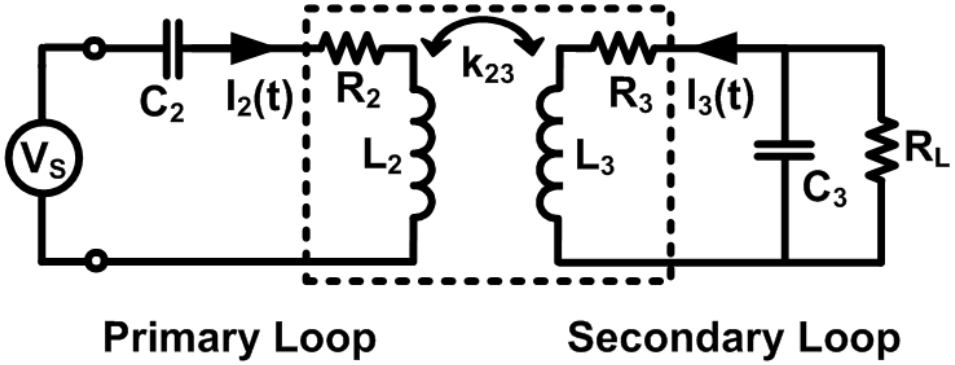




$$\begin{cases} R_{ref} = k_{23}^2 (L_2 / L_3)(Q_3^2 R_3 \| R_L) = k_{23}^2 \omega_0 L_2 Q_{3L} \\ C_{ref} = (L_3 / L_2)(C_3 / k_{23}^2) = 1/(\omega_0^2 L_2 k_{23}^2) \end{cases}$$

Maximizing PTE in 2-Coil Link

$$\eta_{2-coil} = \frac{k_{23}^2 Q_2 Q_{3L}}{1 + k_{23}^2 Q_2 Q_{3L}} \cdot \frac{Q_{3L}}{Q_L}$$



- For a given set of Q_2, Q_3 and k_{23} values, there is an **optimal load**, ***$R_{L,PTE}$***, which can maximize the PTE at that particular arrangement, such as the coupling distance.

$$R_{L,PTE} = \omega_0 L_3 Q_{L,PTE} \qquad Q_{L,PTE} = \frac{Q_3}{(1 + k_{23}^2 Q_2 Q_3)^{1/2}}$$

- PTE is highly dependent on R_L
- R_L is often predefined by the application → **Impedance Transformation**

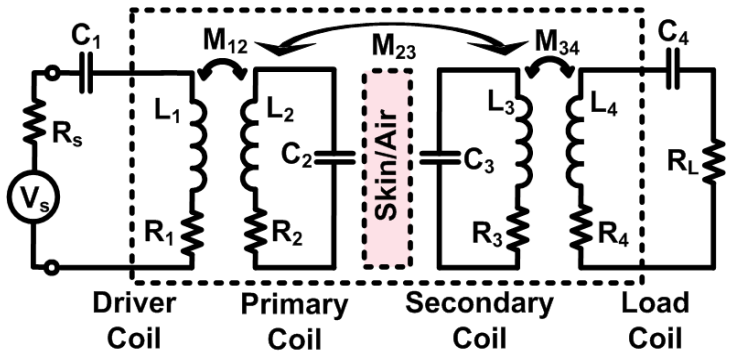
1) Matching circuits 2) **Multiple coils**

PTE and Delivered Power (PDL) in Multi-coil Inductive Links based on RLT

- In an *m*-coil link with negligible coupling between non-neighboring coils
  - R_{ref} from (i+1)th coil to the ith coil: $R_{ref\,i,i+1} = k_{i,i+1}^2 \omega_0 L_i Q_{(i+1)L}$
  - Loaded Q of the ith coil: $Q_{iL} = \omega_0 L_i / (R_i + R_{ref\,i,i+1})$
  - PTE between ith and (i+1)th coils: $\eta_{i,i+1} = R_{ref\,i,i+1} / (R_i + R_{ref\,i,i+1})$
  - *m*-coil link PTE: $\eta_{m-coil} = \prod_{i=1}^{m-1} \eta_{i,i+1} \cdot Q_{mL} / Q_L$
  - PDL: $P_{L,m-coil} = \frac{V_s^2}{2R_1} \frac{1}{1 + k_{12}^2 Q_1 Q_{2L}} \eta_{m-coil}$

Parallel load: $Q_{mL} = R_L / \omega_0 L_m$

Series load: $Q_{mL} = \omega_0 L_m / R_L$



Four-coil link (m = 4)

PTE in Multi-coil Inductive Links based on CMT

- In *m*-capacitively loaded resonators:

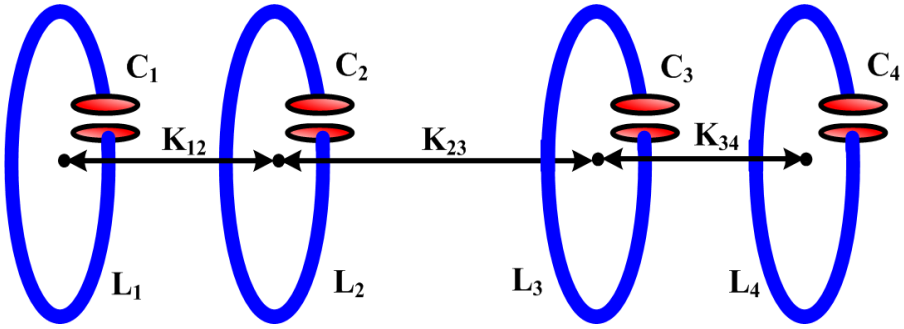
$$\begin{cases} \dfrac{da_1(t)}{dt} = -(j\omega + \Gamma_1)a_1(t) + jK_{12}a_2(t) + F_S(t) \\ \dfrac{da_i(t)}{dt} = -(j\omega + \Gamma_i)a_i(t) + jK_{i-1,i}a_{i-1}(t) + jK_{i,i+1}a_{i+1}(t) \\ \dfrac{da_m(t)}{dt} = -(j\omega + \Gamma_m + \Gamma_L)a_m(t) + jK_{m-1,m}a_{m-1}(t) \end{cases}$$

$$a_i(t) = A_i e^{-j\omega_0 t} \Rightarrow P_i = 2\Gamma_i |A_i|^2$$

$$\eta_{m-coil} = \frac{P_L}{P_S} = \frac{\Gamma_L}{\Gamma_m + \Gamma_L + \sum_{i=1}^{m-1} \Gamma_i \left| \frac{A_i}{A_m} \right|^2}$$

Resonance width: $\Gamma_i = \omega_0 / 2Q_i$

Coupling rate: $K_{i,i+1} = \omega_0 k_{i,i+1} / 2$

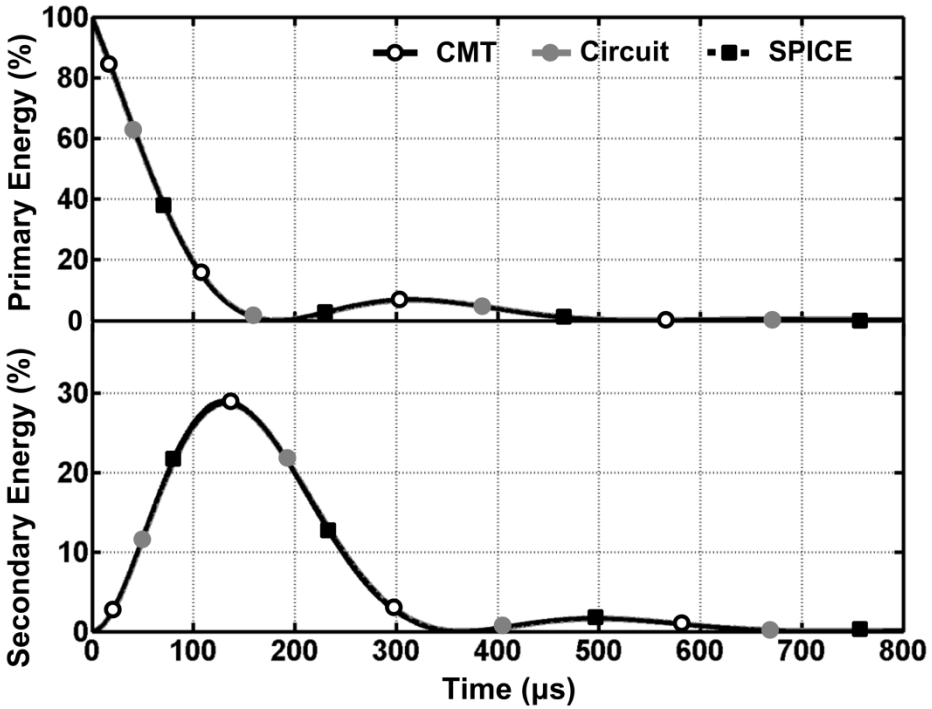


Four-coil link (*m* = 4)

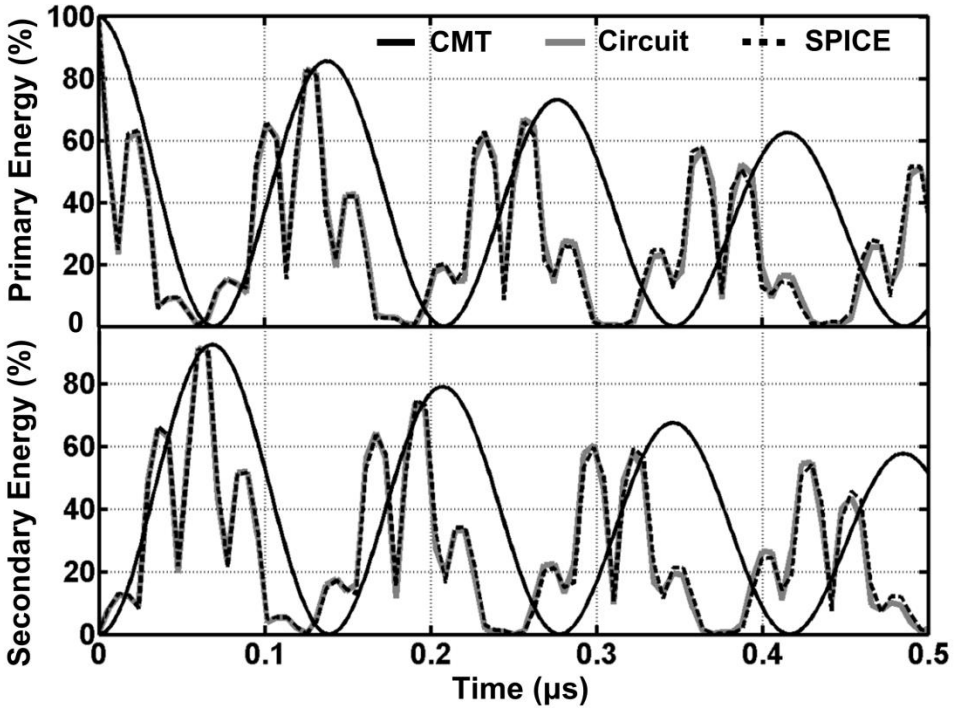
PTE from CMT is identical to the one from RLT!

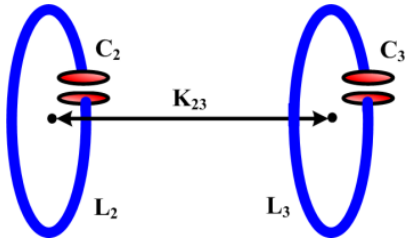
Resonant Magnetic Coupling vs. Inductive Coupling (Transient)

1) Mid-range high-Q condition



2) Short-range low-Q condition



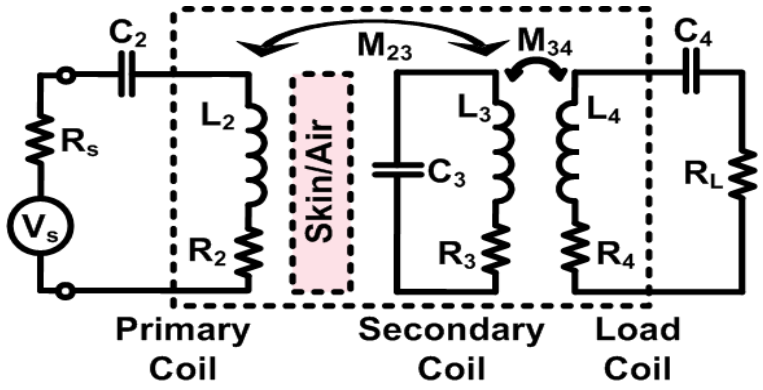


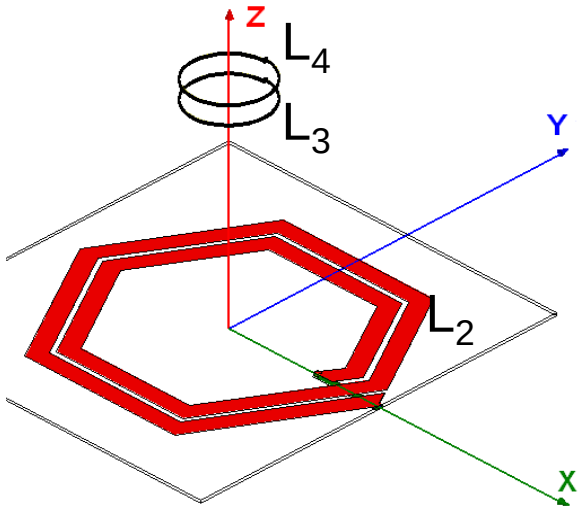
CMT transient response is accurate when coils coupling is weak and coils quality factor is very high!

3-Coil Inductive Link

$$\eta_{3-coil} = \eta_{23}\eta_{34} = \frac{(k_{23}^2 Q_2 Q_3)(k_{34}^2 Q_3 Q_{4L})}{[(1+k_{23}^2 Q_2 Q_3 + k_{34}^2 Q_3 Q_{4L})(1+k_{34}^2 Q_3 Q_{4L})]} \cdot \frac{Q_{4L}}{Q_L}$$

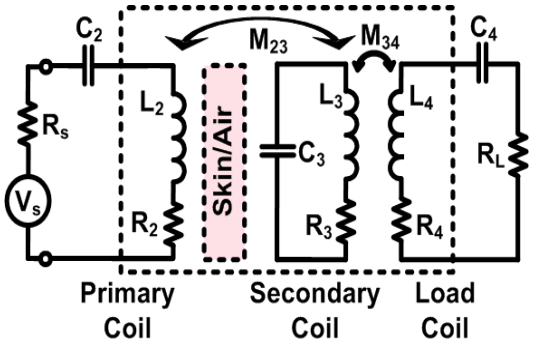
- L_3-L_4 inductive link provides designers with a DoF (M_{34}) to adjust the reflected load on to L_3 to be the optimal value: $R_{L,PTE}$
- Loosely coupled L_2-L_3 link is designed for optimal $R_{L,PTE}$
- Inductive link optimization is decoupled from R_L
- L_3-L_4 inductive link PTE is high as their coupling distance is small



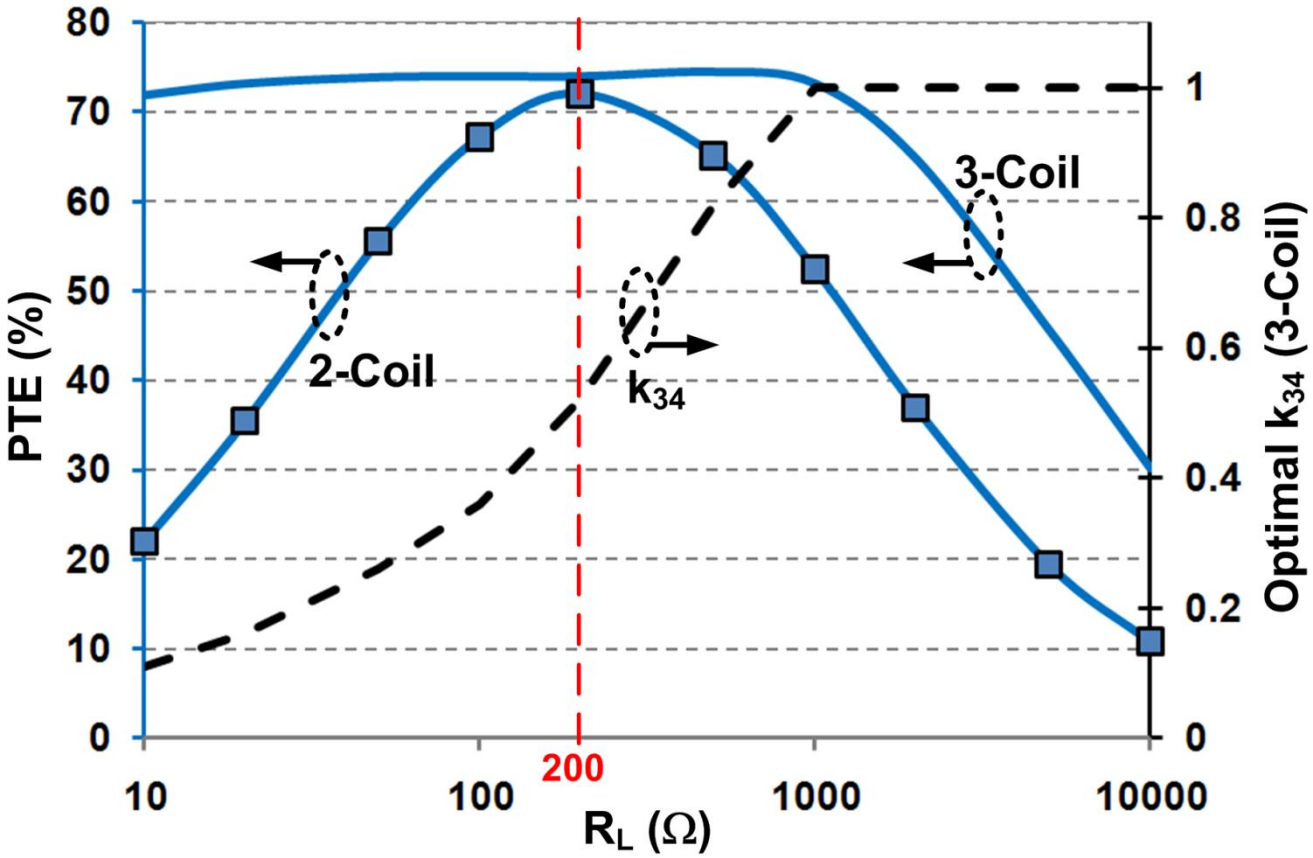


$$R_{L,PTE} = \omega_0 L_3 Q_{L,PTE} \qquad Q_{L,PTE} = \frac{Q_3}{(1+k_{23}^2 Q_2 Q_3)^{1/2}}$$

Maximizing PTE in the 3-Coil Link

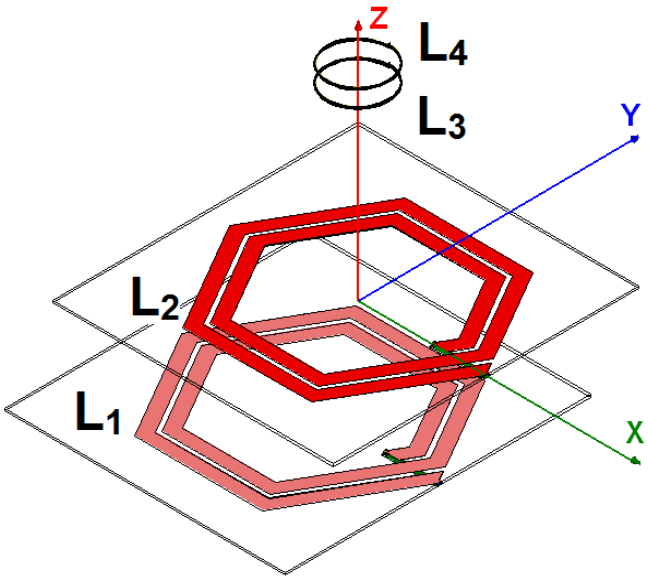


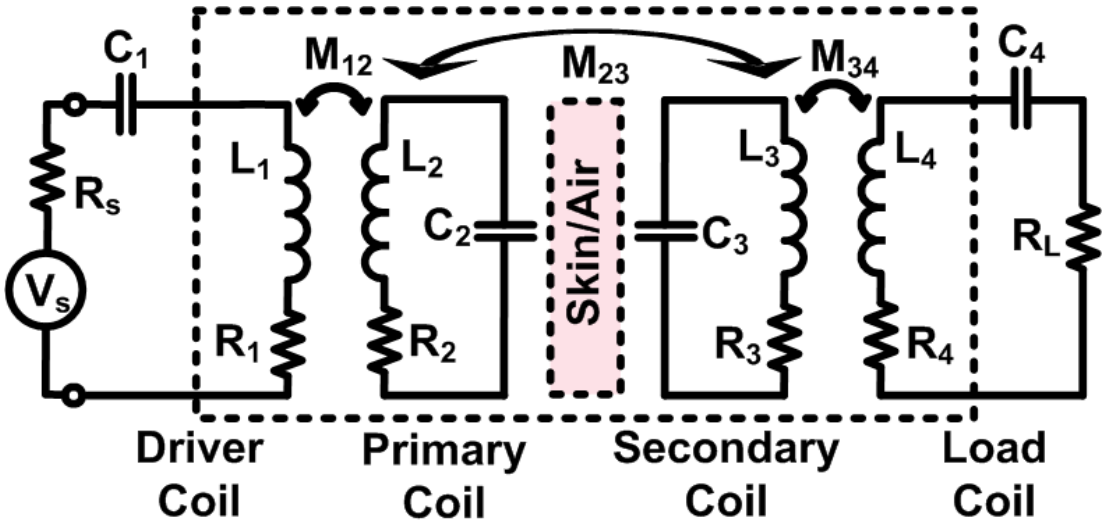
By changing k_{34} (and $R_{ref,3}$), the 3-coil PTE can be kept at maximum for a wide range of R_L.



A 2-coil link does not provide this flexibility, and PTE maximizes only for a specific R_L value.

4-Coil Inductive Link





$$\eta_{4-coil} = \eta_{12}.\eta_{23}.\eta_{34} = \frac{(k_{12}^2 Q_1 Q_2)(k_{23}^2 Q_2 Q_3)(k_{34}^2 Q_3 Q_{4L})}{[(1+k_{12}^2 Q_1 Q_2).(1+k_{34}^2 Q_3 Q_{4L})+k_{23}^2 Q_2 Q_3].[1+k_{23}^2 Q_2 Q_3+k_{34}^2 Q_3 Q_{4L}]} \cdot \frac{Q_{4L}}{Q_L}$$

- 4-Coil link adds an additional DoF for impedance matching on the source side
- If k_{12} is large, the reflected load onto L_1 increases dramatically, which helps maximize the PTE at the cost of reducing PDL

PTE and PDL in 3-Coil Link

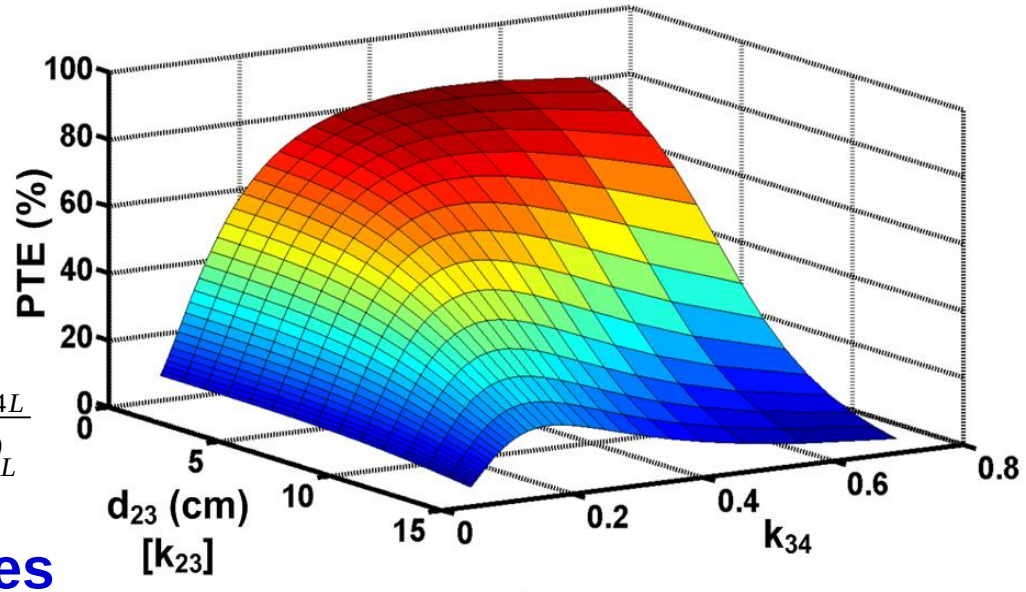
Maximizing PTE should not be at the cost of decreasing PDL

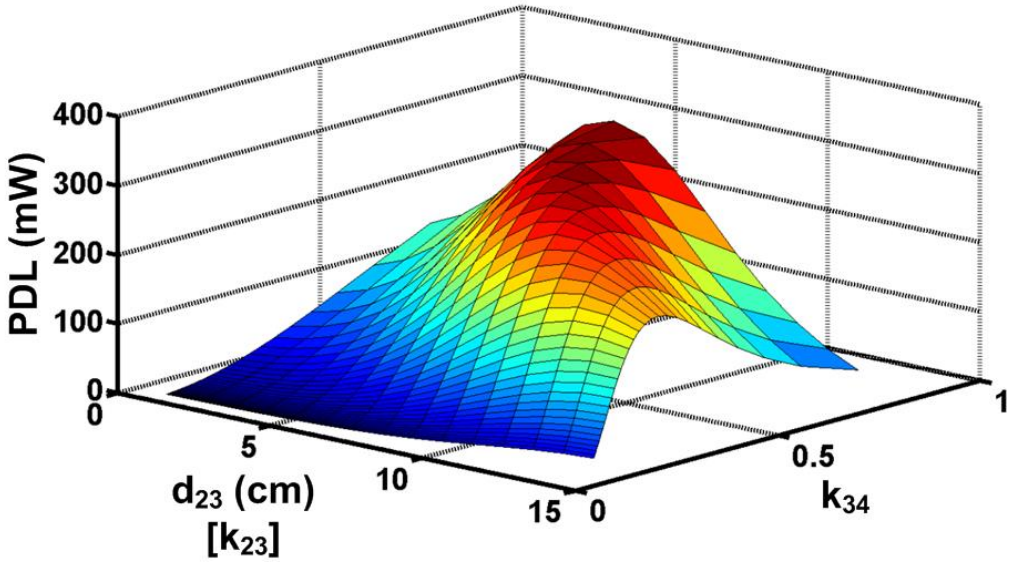
$$P_{L,3-coil} = \frac{V_s^2}{2R_2} \frac{(k_{23}^2 Q_2 Q_3)(k_{34}^2 Q_3 Q_{4L})}{(1 + k_{23}^2 Q_2 Q_3 + k_{34}^2 Q_3 Q_{4L})^2} \cdot \frac{Q_{4L}}{Q_L}$$

The optimal design maximizes both PTE and PDL

$$k_{23,PDL} = \left(\frac{1 + k_{34}^2 Q_3 Q_{4L}}{Q_2 Q_3} \right)^{1/2}$$

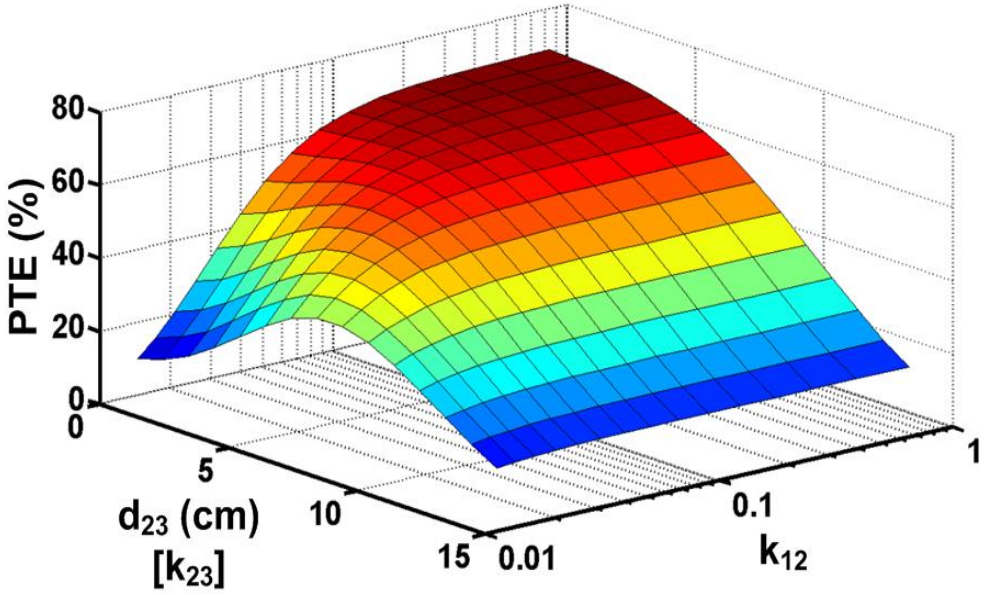
$$k_{34,PDL} = \left(\frac{1 + k_{23}^2 Q_2 Q_3}{Q_3 Q_{4L}} \right)^{1/2}$$

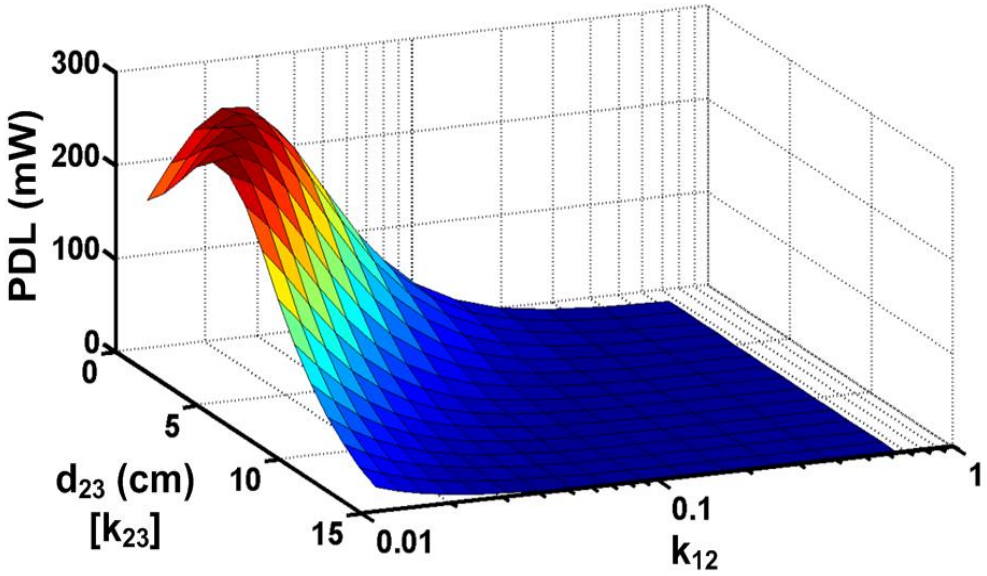




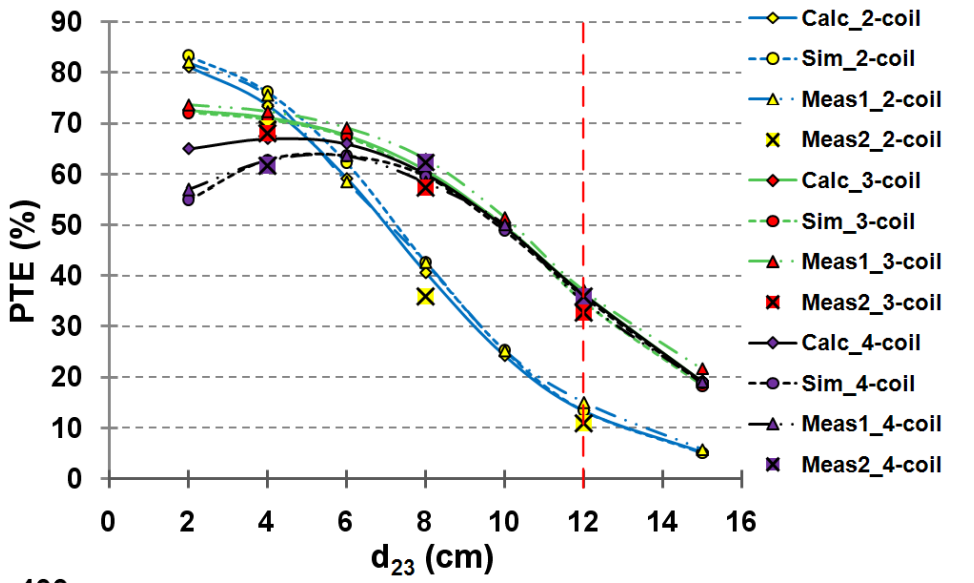
PTE and PDL in 4-Coil Link

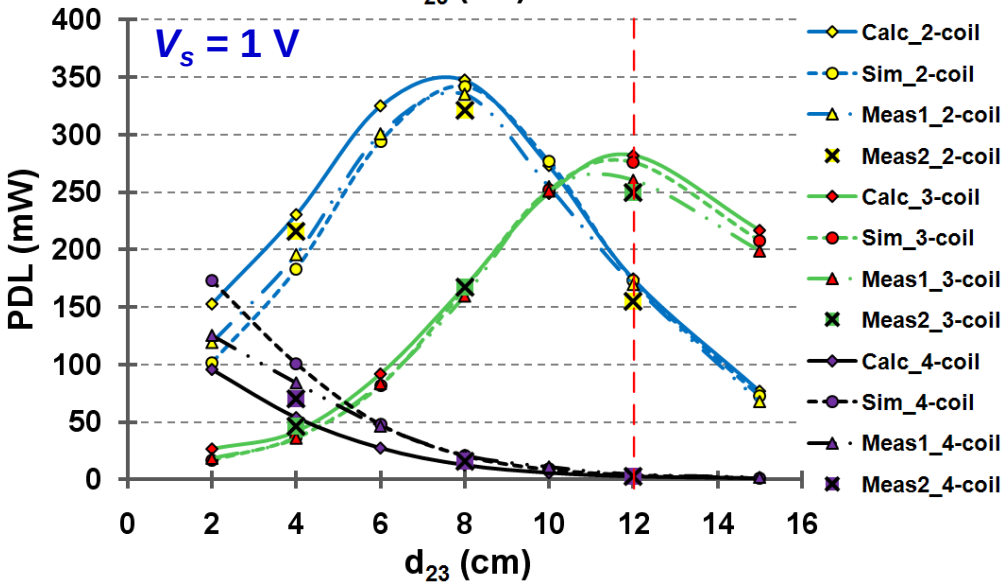
- If k_{12} is large enough, 4-coil can tolerate variations in coil separation (k_{23}) and maintain a large PTE
- Large k_{12} reduces the available power from the source
- **Small overlap between high PTE and PDL areas**

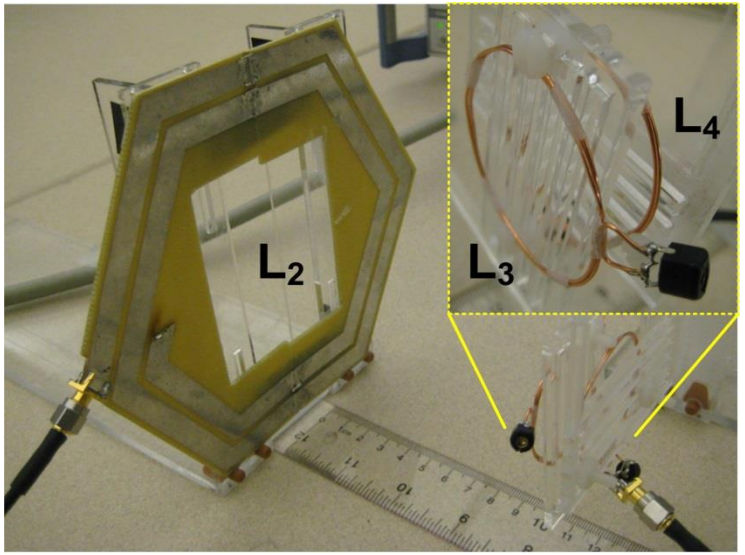




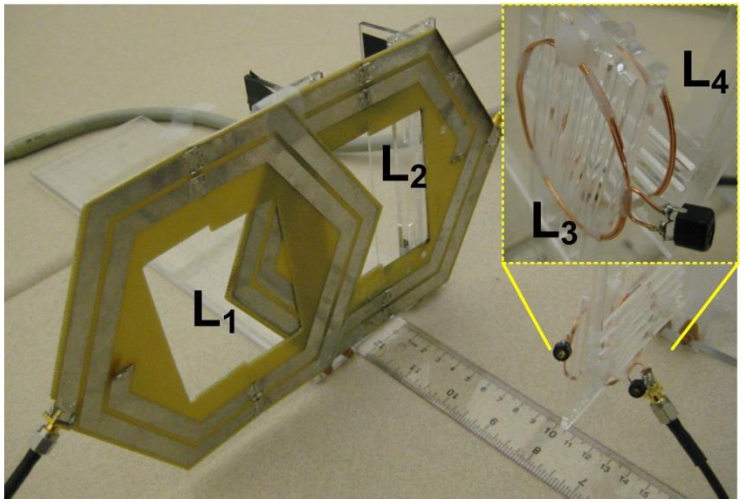
3-Coil vs. 4-Coil Link (Measurements)





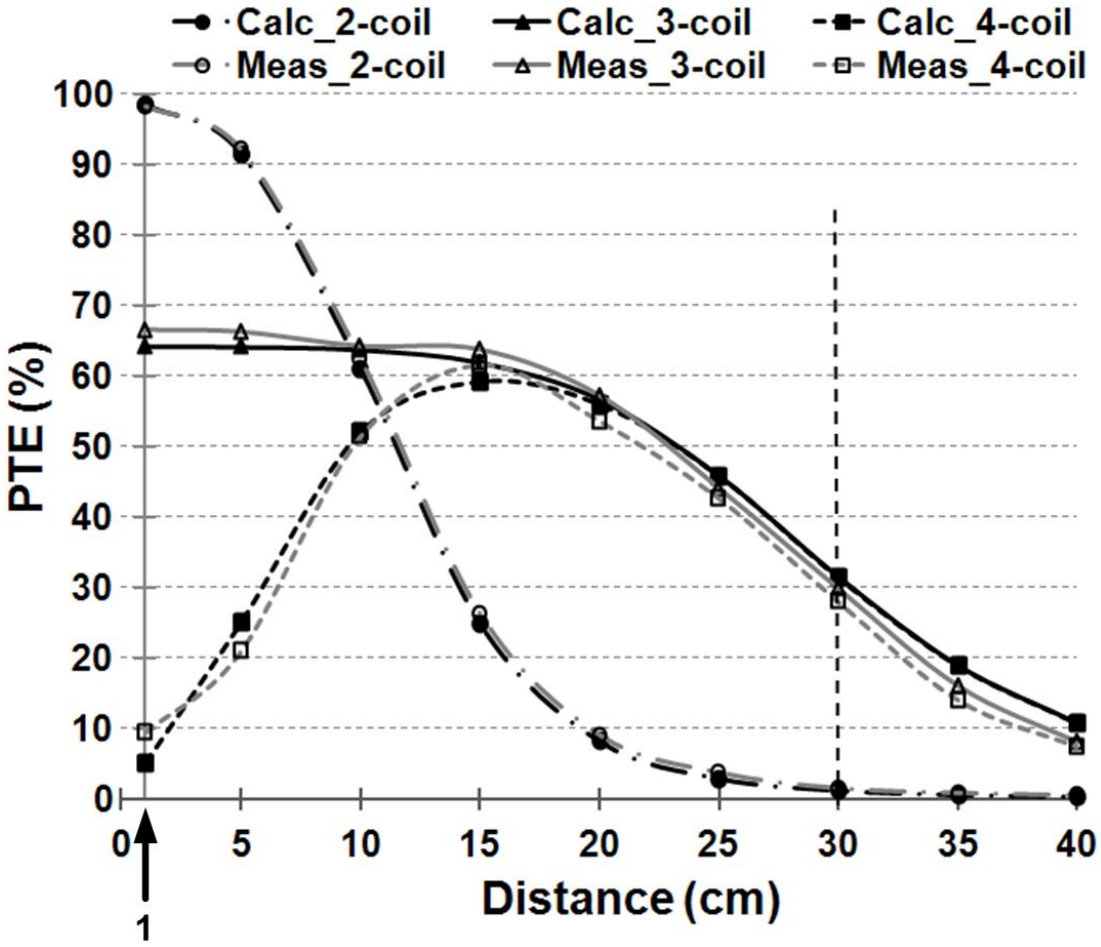


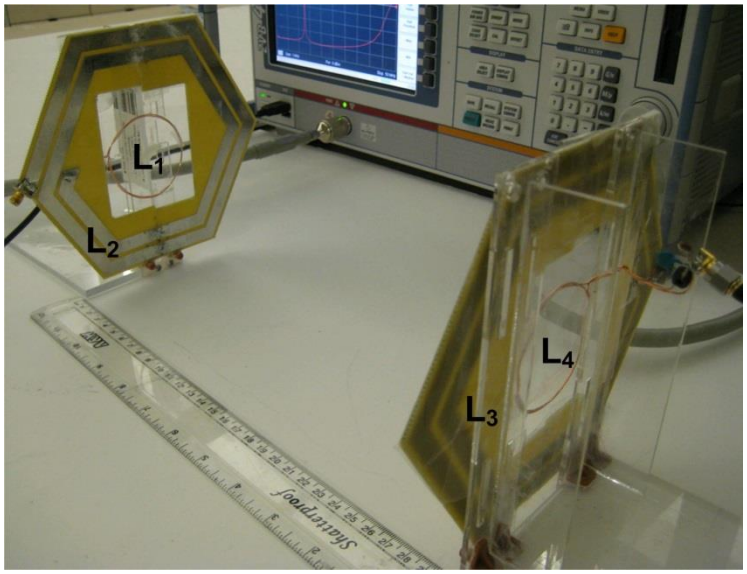
3-coil link (PDL = 260 mW)



4-coil link (PDL = 4.4 mW)

Resonant-Magnetic Coupling vs. Inductive Coupling (PTE)





Four-coil link measurement setup. Three and two-coil links used similar coils except L_1 and L_4, respectively.

Calculated PTEs were similar from CMT and RLT and matched very well with measurements!

A New Figure of Merit (FoM) for Inductive Power Transmission

- Conventional design merits are either **PTE** or **PDL**
- How to balance PTE and PDL?
- How to choose between 2-, 3-, and 4-coil links?

$$FoM = \frac{\eta_{m-coil}^{n} \times P_{L,m-coil}}{V_s^2}$$

***n*: PTE weight, which depends on the application**

- FoM effect on PTE and PDL drop:

$$\eta_{Loss} = \frac{\eta_{\max} - \eta_{FoM}}{\eta_{\max}} = \frac{1}{n+2},$$

$$P_{L,Loss} = \frac{P_{L,\max} - P_{L,FoM}}{P_{L,\max}} = \frac{n^2}{(n+2)^2}$$

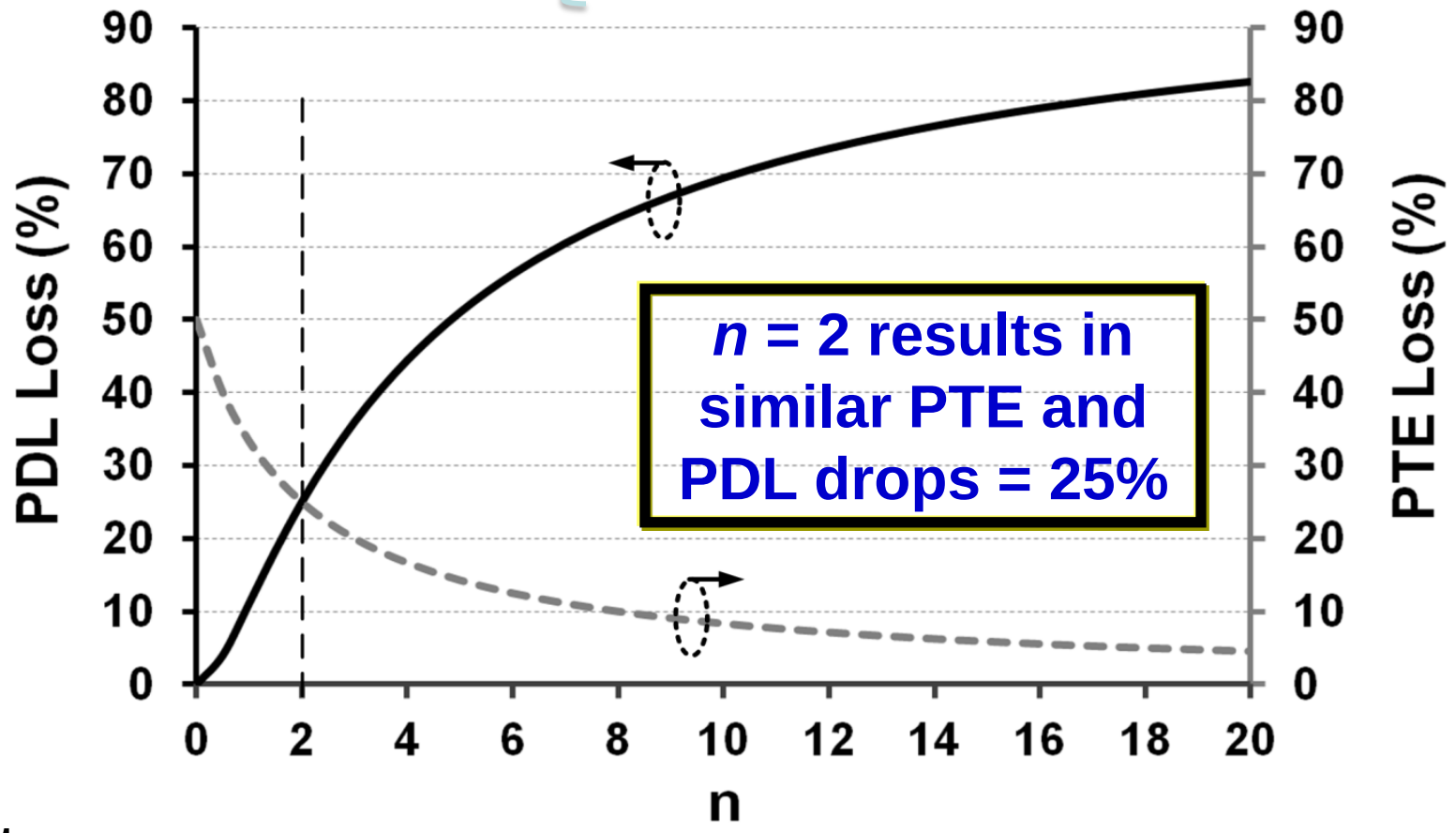
$\eta_{\max}, P_{L,\max}$: PTE and PDL are maximized

$\eta_{FoM}, P_{L,FoM}$: FoM is maximized

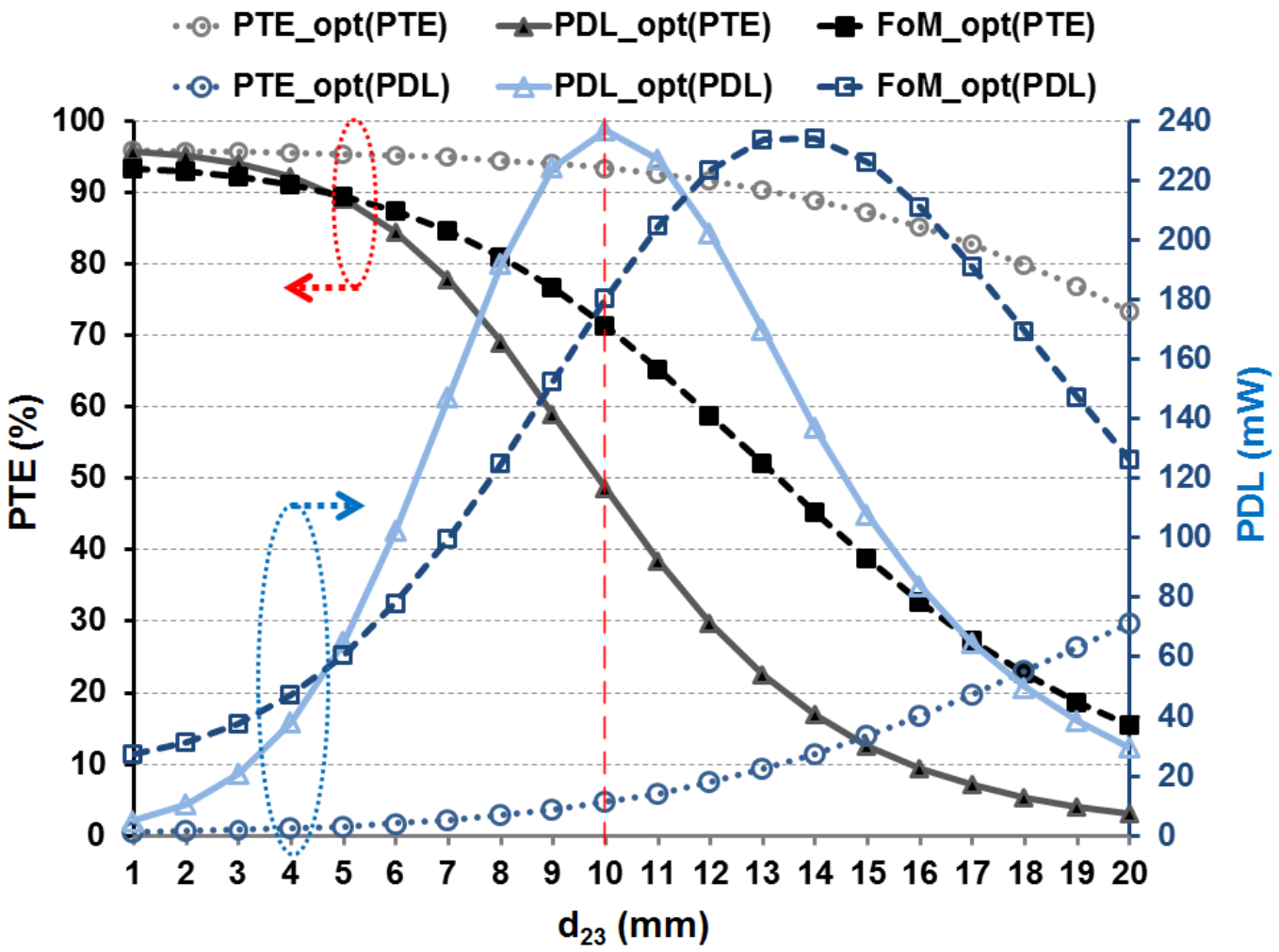
PTE vs. PDL Loss based on FoM for 2-, 3-, and 4-coil Links

$$FoM = \frac{\eta_{m-coil}^{n} \times P_{L,m-coil}}{V_s^2}$$

- $n = 0 \Rightarrow FoM \sim PDL$
- $n \to \infty \Rightarrow FoM \sim PTE$



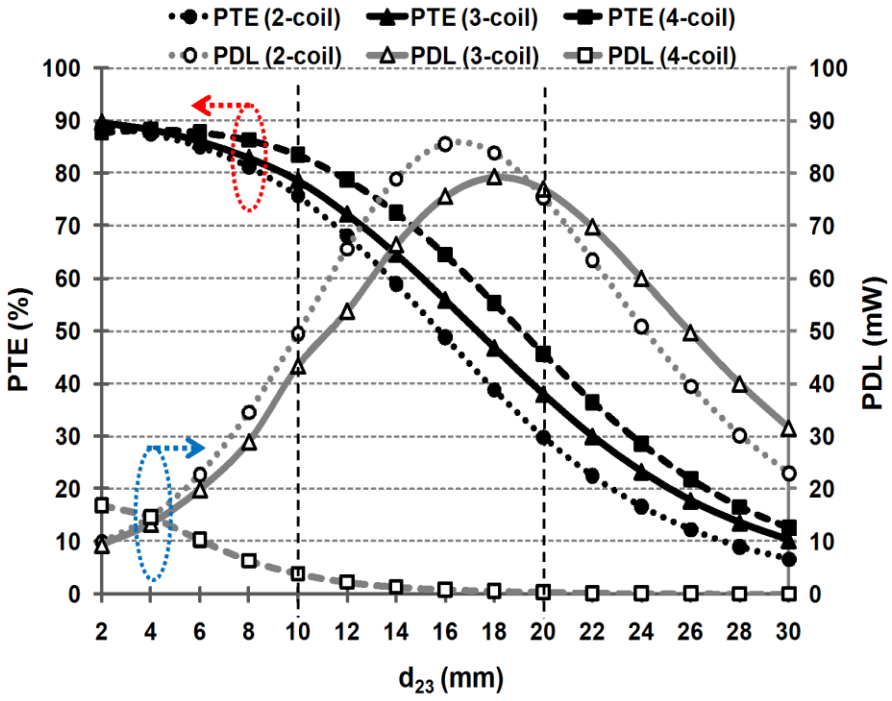
Optimal 2-coil Link for IMDs based on FoM

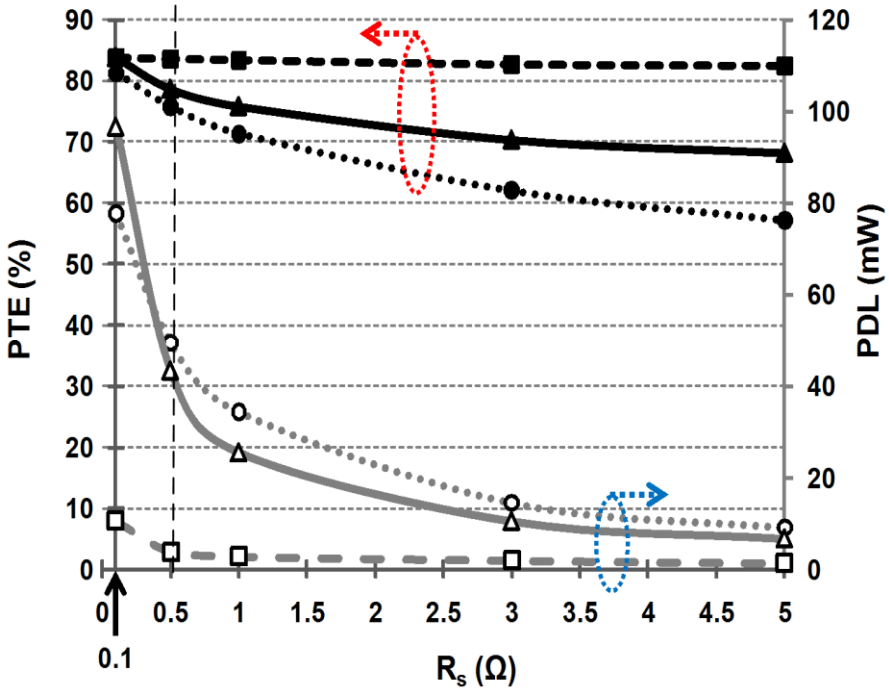


- 2-coil Link for IMDs
- Rx coil dia. = 10 mm
- V_s = 1 V
- R_s = 0.5 Ω
- R_L = 100 Ω
- f_0 = 13.56 MHz
- d_{23} = 10 mm

At d_{23} = 10 mm, FoM-optimized link provides 47% more PTE than the PDL-optimized link and 16 times larger PDL than the PTE-optimized link.

Comparing 2- 3- & 4-Coil Links for IMDs

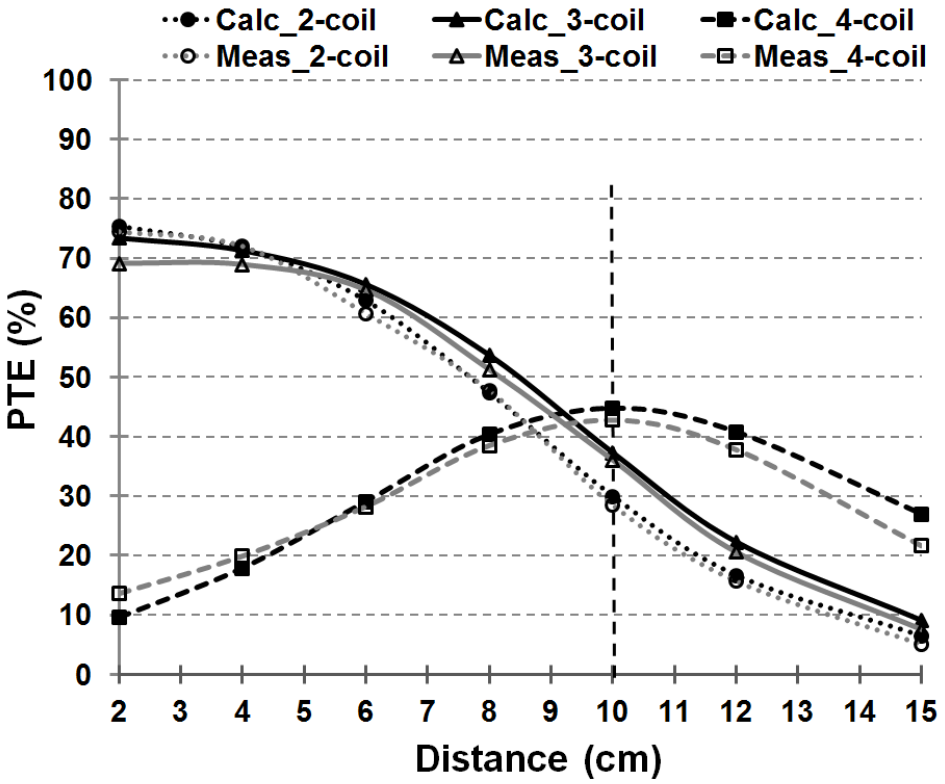


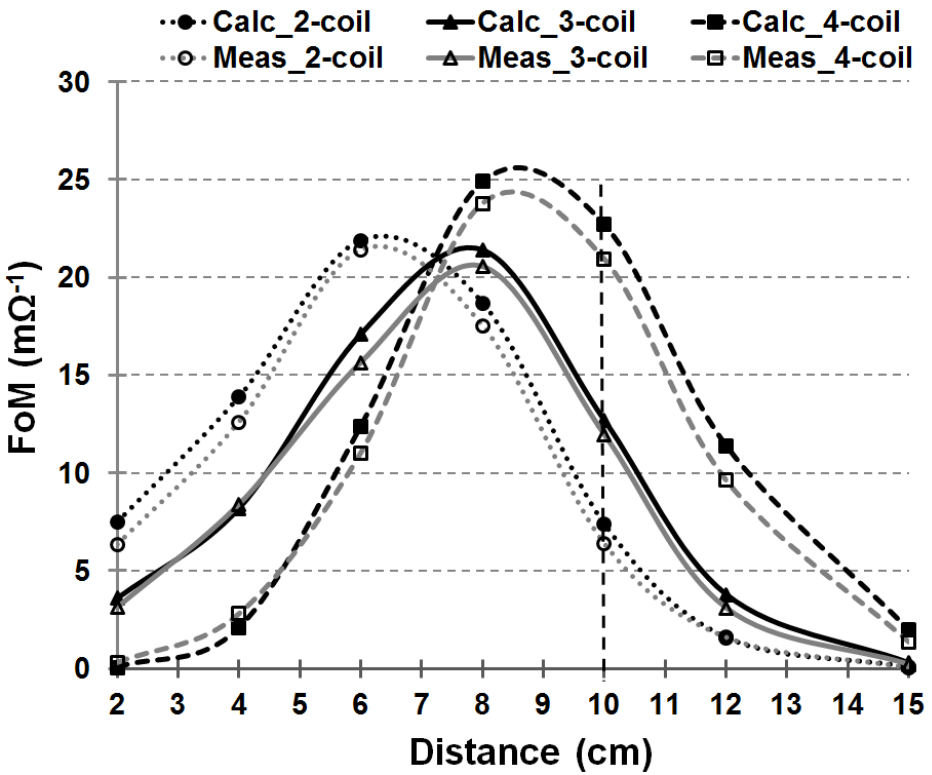


- At d_{23} = 10 mm, 4-coil link provides 4.9% more PTE and 11 times less PDL than an equivalent 3-coil. At d_{23} = 20 mm, PTE difference is 7.7%, while PTE of 3-coil is 192 times larger than an equivalent 4-coil
- For large R_s, 4-coil is optimal

An FoM including both PTE and PDL is needed to differentiate between 2-,3-, and 4-coil links!

Optimal Multi-coil Link for Charging Handheld Mobile Devices based on FoM





The 4-coil link has superior FoM at d_{23} = 10 cm at the cost of much lower PTE, and consequently the FoM, at shorter coupling distances.

Rx coil dia. = 4 cm, R_s = 0.5 Ω, R_L = 5 Ω, f_0 = 13.56 MHz, d_{23} = 10 cm

Optimal Multi-coil Link based on FoM

	2-Coil Link	3-Coil Link	4-Coil Link
Strong coupling (*k*)	☺	☺	☹
Weak coupling (*k*)	☹	☺	☺
Large PDL (small R_s)	☺	☺	☹
Small PDL (large R_s)	☹ ☹	☹	☺
Coupling variations & small R_s	☺	☺ ☺	☹
Coupling variations & large R_s	☹	☺	☺ ☺